Reciprocal interactions out of congestion-free adaptive networks

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(Received 1 November 2011; published 21 February 2012)

In this paper we study the jamming transition in complex adaptive networks. We introduce an adaptation mechanism that modifies the weight of the communication channels in order to delay the congestion onset. Adaptivity takes place locally as it is driven by each node of the network. Surprisingly, regardless of the structural properties of the backbone topology, e.g., its degree distribution, the adaptive network can reach optimal functioning provided it allows a reciprocal distribution of the weights. Under this condition, the optimal functioning is achieved through an extensive network reshaping ending up in a highly reciprocal weighted network whose critical onset of congestion is delayed up to its maximum possible value. We also show that, for a given network, the reciprocal weighting obtained from adaptation produce optimal static configurations.

DOI: 10.1103/PhysRevE.85.026112 PACS number(s): 89.75.Hc, 05.70.Fh, 89.75.Fb

I. INTRODUCTION

In recent years, the use of complex network theory [1] together with tools inherited from nonequilibrium statistical physics have allowed us to study the critical properties of a variety of complex systems. These critical properties are of utmost importance to describe the functionality of networked systems in practical terms such as their resilience to failures, attacks, and the spreading of diseases [2,3] or their ability to attain synchronized dynamics [4] and display collective behaviors in social systems [5], among others. It has been shown that the structure of interactions among the constituents of real complex systems influences dramatically the above properties due to its scale-free (SF) nature [6], revealed from the power-law probability distribution of the number of contacts (degree), k, per individual: $P(k) \sim k^{-\gamma}$.

In the same way, the structure of communication systems, such as the Internet [7], affects their dynamical properties and the onset of collective states, such as jamming and congestion, that are detrimental to their functioning. The architecture of these systems is described by means of a collection of N nodes and L edges that make up a macroscopic network within which packets travel from their corresponding departure nodes to the arrival ones. The path followed by each packet is described by the sequence of nodes visited along its trip. However, the nodes have a limited capacity for receiving, allocating, or sending packets. Thus, congestion eventually appears when nodes are not able to balance their incoming and outgoing flows of packets. These microscopic imbalances give rise to a macroscopic jammed state in which the number of packets traveling across the network grows in time.

The problem of congestion becomes of utmost importance in real communication networks due to their SF nature [7]. The presence of largely connected nodes, the *hubs*, having a large number of communication channels departing from and arriving to them, anticipates considerably the onset of congestion in such a way that the network is very prone to fail, even when a low quantity of packets are injected into the network per unit time. Thus, a large body of recent work has been focused on the formulation of different strategies aimed at delaying the critical onset of jamming [8–25] to increase the traffic capacity of the networks. However, despite the advances in this direction, achieving optimal functioning in SF architectures still constitutes a major challenge.

In this work, we approach the problem of delaying jammed states from the perspective of adaptive networks [26], which has been successfully used as a way for enhancing the functionality of network topologies in diverse settings such as synchronization [27–31], sustainability of cooperation in evolutionary games [33], or resilience to epidemics [32]. In our case, the network topology of the communication system will adapt its connections in order to avoid congestion. In particular, we will consider that each node of the network will adapt the weights of its outgoing links to distribute its deliveries preferentially to its less congested neighbors. Thus, the adaptive principle makes use of purely local information. Surprisingly, this simple adaptive network is able to delay the congestion onset up to the optimal one regardless of the heterogeneity of the network of physical links (the network backbone). Furthermore, as a product of the adaptive process we obtain optimal weighted networks that show highly reciprocal patterns for the interaction between connected elements. Finally, we show that these reciprocal topologies obtained from local adaptation, when used as static topologies, behave optimally. Therefore, our results show adaptivity as an efficient way to reach optimal functioning in communication models while the reciprocal design, showing up from adaptation, appears near the optimal architecture for any network substrate.

II. THE MODEL

We start our analysis by considering a simplified routing model, although we will later show the validity of our results in a more realistic scenario. The model works as follows. At each time step, each node i of the network creates a new packet with

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probability p and stores it in its queue, q_i. Thus, on average $p \cdot N$ new packets are created at each time step. Additionally, at each time step, each node i of the network, provided it has packets waiting in its queue, delivers one of the packets to one if its k_i neighbors. To decide where to send the packet, each link from i to a neighbor j has assigned a weight W_{ij} that represents the probability of sending a packet through the channel $i \to j$. Thus, we model the dynamics of each packet as a discrete walk process on top of the network, so the weight of the links departing from each node i fulfill $\sum_j W_{ij} = 1$ while this is not necessarily true for the total strength of its incoming links: $s_i = \sum_j W_{ji}$. Finally, with probability μ, a packet hopping from node i to a neighbor j reaches its destination and it is destroyed, otherwise, with probability $(1-\mu)$, it is enqueued in j.

Simultaneously with the traffic dynamics, the network adapts the weights of its links according to the level of congestion. This adaptation takes place locally so, at each time step, each node i assigns the weights of its outgoing links depending on the congestion level of its neighborhood according to

$$W_{ij}^{t+1} = \frac{A_{ij}\mathrm{e}^{-\beta q_j^t}}{\sum_{l=1}^{N} A_{il}\mathrm{e}^{-\beta q_l^t}} , \qquad (1)$$

where q_j^t is the queue length of a node j at time step t and A_{ij} is the (i,j) element of the adjacency matrix, $\mathbf{A}$, defined as $A_{ij} = 1$ when there is a link between i and j while $A_{ij} = 0$ otherwise. In the above equation the parameter β accounts of the sensitivity of nodes to their neighbors' congestion. Note that the above assignation preserves the normalization of the outgoing links $\sum_j W_{ij}^t = 1\ \forall i,t$. Obviously, when $\beta = 0$ network adaptation does not occur, and we recover a random walk dynamics for the packets' motion.

The numerical simulation of the above adaptive network model allows us to study the effect of the load of traffic, p, and the strength of adaptation, β, on the level of congestion of the system. For fixed values of p and β, the macroscopic state of the system will be characterized by means of the order parameter ρ_t introduced in Ref. [34] that accounts of the rate of growth of the active packets, $P(t)$, in the network during a time window of duration τ relative to the external flow during the same period $(Np\tau)$:

$$\rho_t = \lim_{\tau \to +\infty} \frac{P(t+\tau) - P(t)}{Np\tau} . \qquad (2)$$

The average value of ρ_t, ρ, describes the congestion state of the system. Obviously, when the system is in the free-flow (or undercongested) phase we have $\rho = 0$, since the network is able to balance the incoming flow with the delivery of active packets. On the contrary, when $\rho > 0$ the congestion of the system grows in time (with rate ρ) attaining its maximum imbalance when $\rho = 1$. For a fixed value of β, a phase transition occurs at some critical value p_c separating the free-flow phase $(p < p_c)$ from the congested one $(p > p_c)$.

Before describing the congestion levels reached by the adaptive network, let us derive the optimal critical point, p_c^*, and the optimal weight topology, $\{W_{ij}^*\}$, of any network operating with the routing dynamics described above. To this end, we write the evolution of the queue length of nodes as

$$q_i^{t+1} = q_i^t + p + \sum_{j=1}^{N} W_{ji}^t \theta\left(q_j^t\right)(1-\mu) - \sum_{j=1}^{N} W_{ij}^t \theta\left(q_i^t\right), \qquad (3)$$

where $\theta(x) = 1$ when $x > 0$ and $\theta(x) = 0$ otherwise. Equations (1) and (3) define the evolution of the adaptive network.

We, first, assume that the dynamics ends up in a stationary state of the free-flow regime $(p \leqslant p_c)$, i.e., $q_i^{t+1} = q_i^t\ \forall i$. From Eq. (1) it is clear that this assumption automatically implies that the links' weights are also stationary, $W_{ij}^{t+1} = W_{ij}^t\ \forall i,j$. Second, we assume that all the nodes are occupied by at least one packet, i.e, the queues are nonzero: $\theta(q_i) = 1\ \forall i$. This second hypothesis holds for $p \geqslant p_c$. Consequently, if the two hypothesis hold at once, the system is situated at the onset of congestion, $p = p_c$. More importantly, these two conditions impose that both the stationary distribution of weights and the critical load ($\{W_{ij}^*\}$ and p_c^*) are optimal, since all the nodes become congested at once at the onset of jamming. Thus, imposing the two conditions for optimal functioning in Eq. (3) we obtain

$$p_c^* + \sum_{j=1}^{N} W_{ji}^*(1-\mu) = \sum_{j=1}^{N} W_{ij}^*. \qquad (4)$$

Considering the normalization of the outgoing weights, $\sum_{j=1}^{N} W_{ij}^* = 1$, we obtain that the incoming strength of each node in the optimal configuration, $s_i^* = \sum_j W_{ji}^*$, is

$$s_i^* = \frac{1-p_c^*}{1-\mu} \quad \forall i, \qquad (5)$$

at the critical point p_c^*. On the other hand, since we are in the stationary state, the total incoming flow, $\sum_i s_i$ has to be equal to the outgoing one which, in its turn, is equal to $\sum_{i,j=1}^{N} W_{ij}^* = N$ since, at p_c^*, all the nodes are occupied. Thus, at p_c^*, the incoming strength of nodes is $s_i^* = 1\ \forall i$ and, from Eq. (5), we obtain that the congestion onset reaches the optimal (maximum possible) value: $p_c^* = \mu$.

Let us note that we have not imposed any particular structure to the adjacency matrix $\mathbf{A}$. Therefore, in principle, for a given network backbone $\mathbf{A}$, optimal behavior can be attained provided it is possible to achieve a weight distribution $\{W_{ij}\}$ such that $s_i = 1\ \forall i$. In the following we will show that the proposed adaptive scheme is able to reshape the weight structure of the networks in order to achieve optimality. To this aim, we have carried out numerical experiments by starting from an empty network, i.e., with empty queues for all the nodes, and with the following initial configuration for the links' weights: $W_{ij}^0 = 1/k_i$. We then let the traffic and topology coevolve until the value of ρ reaches a stationary value.

The backbone topologies, encoded in $\mathbf{A}$, used along the work are Erdös-Rényi (ER) random graphs (having a Poissonian degree distribution) and SF networks. In both cases, we generated them by means of a model introduced in Ref. [35] that interpolates between SF networks with a degree distribution $P(k) \sim k^{-3}$ and ER graphs. In addition, to generate SF networks with different exponents γ in their degree distribution, we have used the configurational model [36]. In all cases, we take care that the networks are connected, i.e., they have a single connected component.

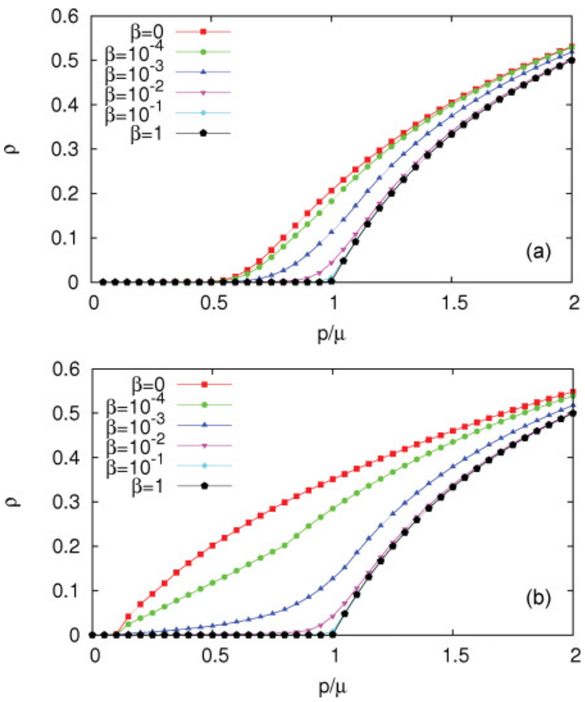

FIG. 1. (Color online) Congestion diagrams $\rho(p/\mu)$ for (a) ER and (b) SF ($\gamma = 3$) networks. Both topologies have $N = 5000$ nodes and an average degree of $< k >= 8$. In the two panels the diagrams correspond to $\beta = 0$ (no adaptivity), $\beta = 10^{-3}$ (slow adaptive dynamics), and $\beta = 1$ (fast adaptive dynamics). The points show the results obtained through numerical simulations of the routing dynamics (50 realizations per point) while the curvs are for the solution of the associated theoretical model, Eqs. (1) and (3). In both cases the arrival-to-destination probability is set to $\mu = 0.2$ as the usual value found in the Internet [7].

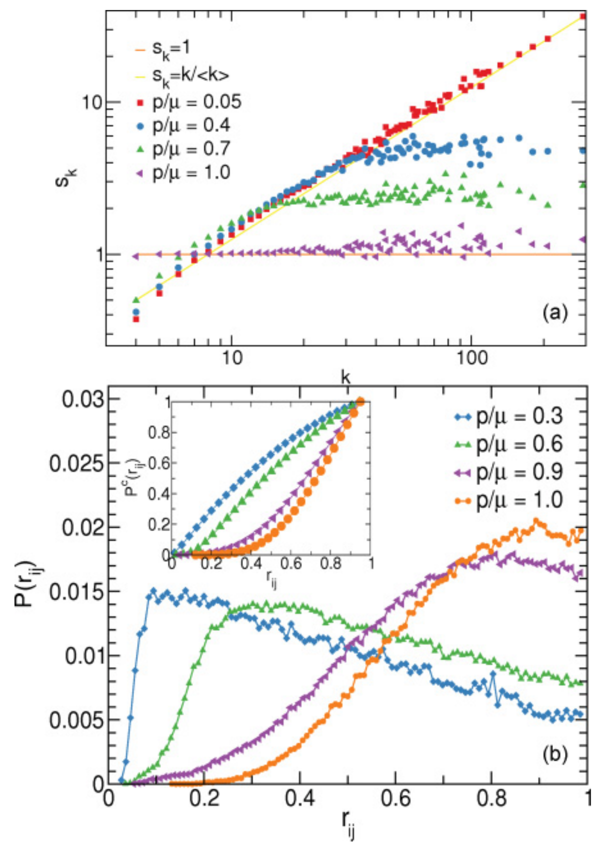

FIG. 2. (Color online) The panels show (a) the incoming strength of nodes as a function of their degree, s_k and (b) the probability density function for link reciprocity, r_{ij}, as defined in Eq. (6) for different values of p/μ. Additionally, the inset in (b) shows the cumulative distribution for the reciprocity of links. In both cases, we used $\mu = 0.2$ and a SF ($\gamma = 3$) backbone with $N = 5000$ and $\langle k \rangle = 8$.

III. RESULTS

We start our analysis by measuring the value of p_c that separates the congested and the free-flow regimes for ER and SF networks topologies when there is no adaptivity in the system ($\beta = 0$). In Fig. 1(a) we observe that for ER graphs congestion starts when $p_c/\mu \sim 0.75$ while for SF networks [Fig. 1(b)] congestion occurs much earlier and $p_c/\mu \sim 0.1$. However, when we let the system self-adapt, the weights of the links by increasing the value of β, the congestion point for both networks shifts toward the optimal onset and finally, for $\beta = 1$, the critical load is optimal, $p_c = p_c^* = \mu$. The results indicate that the adaptive topology is able to find a distribution of weights that optimizes the traffic distribution across the network, independently of the underlying interaction backbone.

To understand the optimal functioning of the adaptive networks, we analyze the system at a microscopic level. In particular, we measure the effect of β in (i) the incoming strength of the nodes, s_i, and (ii) the link reciprocity, r_{ij}, that quantifies the relation between the values of the weights, W_{ij} and W_{ji}, associated to each link (i,j) as

$$r_{ij} = \frac{\min[W_{ij},W_{ji}]}{\max[W_{ij},W_{ji}]} . \qquad (6)$$

Obviously, when $r_{ij} = 1$, the link connecting nodes i and j is fully reciprocal while $r_{ij} \to 0$ as reciprocity decreases.

In Fig. 2(a) we show the strength of nodes as a function of their corresponding degree for SF networks and different values of p/μ corresponding to the subcritical (free-flow) regime and the optimal critical point, $p_c^* = \mu$. We observe that, when the load of traffic is low (e.g., $p/\mu = 0.05$), the network adaptivity causes almost no change and the adaptive dynamics ends up almost in the original wiring $\{W_{ij}^0 = 1/k_i\}\ \forall\, i,j$. Thus, the incoming strength scales linearly with the degree of the node, $s_k = k \sum_{k'} P(k'|k)/k' \simeq k \sum_{k'} P(k')/\langle k \rangle = k/\langle k \rangle$, so hubs are prone to receive the largest number of packets. However, as traffic load p increases, adaptivity progressively homogenizes the incoming strength of the nodes so, at the optimal critical point, p_c^*, $s_i = 1\ \forall i$, in agreement with the results obtained for the wiring in the optimal critical point of the Markovian model [Eqs. (1) and (3)].

Since many possible microscopic configuration may satisfy the condition $s_i = 1\ \forall i$, the question naturally arises with regard to the precise mechanism by which adaptivity reaches optimality. One possible solution is a fully reciprocal configuration $W_{ij} = W_{ji}\ \forall\, i,j$ since in such a situation $s_i = \sum_j W_{ji} = \sum_j W_{ij} = 1\ \forall\, i$. In Fig. 2(b) we present the probability density function of the values r_{ij}, $P(r_{ij})$, measured

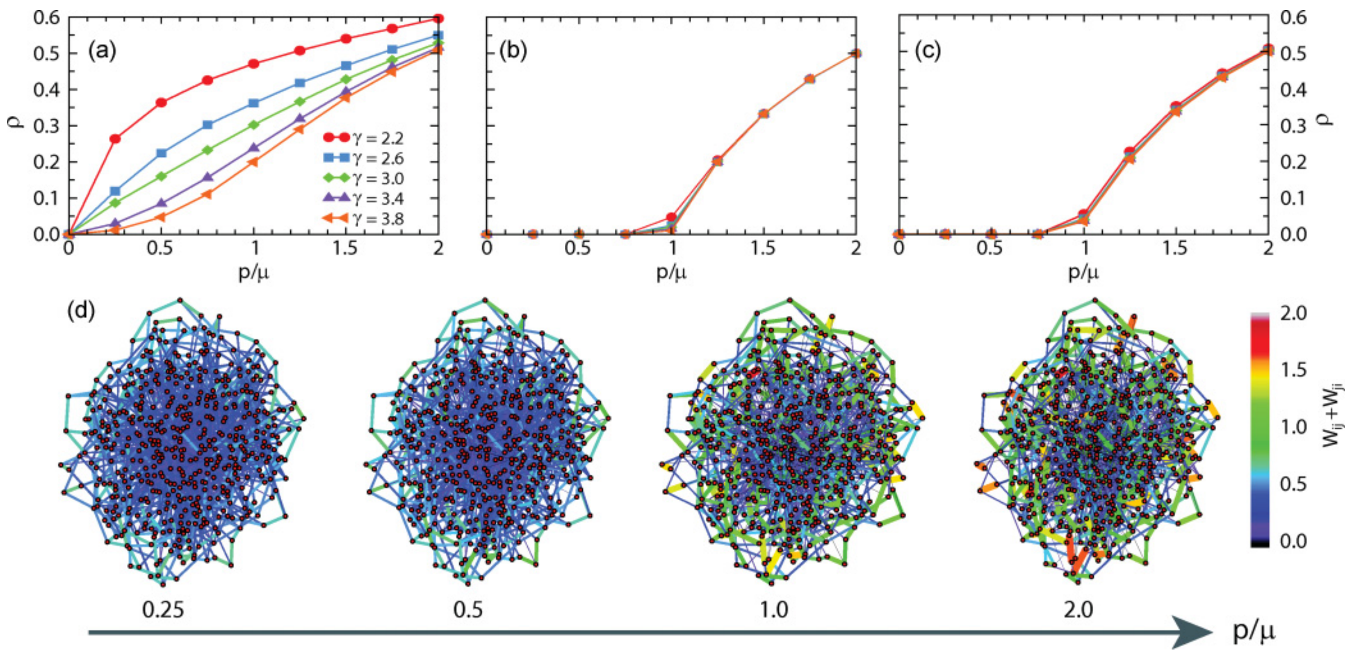

FIG. 3. (Color online) Congestion diagrams $\rho(p/\mu)$ for SF networks of size $N = 500$ with different exponents γ. In (a) we set $\beta = 0.0$ so no adaptation occurs. As a product, we observe large levels of congestion even for low values of p. However, when adaptivity is at work (b), congestion decreases dramatically and all the adaptive networks reach the optimal critical value $p_c^* = \mu$. In (c) we fixed the weights of the links to the values obtained at the optimal critical point after applying the adaptive process. We then run the traffic dynamics with no adaptation ($\beta = 0.0$) for the whole range of p/μ values reaching almost optimal congestion values such as those shown in (b). The networks below (d) show the weighted topologies obtained from the adaptive dynamics for different values of p/μ. The total (incoming plus outgoing) weight of each link corresponds to the color code in the right. In all the panels the results correspond to fixing $\mu = 0.2$.

in the adaptive networks produced for different values of p/μ corresponding to the free-flow regime and the optimal critical onset. From the figure we observe that when the system is still far from the congestion point (e.g., $p/\mu = 0.3$), the values of the links are far from reciprocity and only around 30% have a value of $r_{ij} \geqslant 0.5$. However, as we approach the critical point, reciprocity starts to increase reaching, at $p^* = \mu$, with roughly 90% of the links having $r_{ij} \geqslant 0.5$. Thus, it seems clear that reciprocity provides adaptivity with a successful route toward optimality.

As a by-product of the adaptive process, we analyze the behavior of the optimal configuration of weights obtained from adaptation at $p/\mu = 1$ as static structures for the whole range of p/μ values. First, we generate SF networks with different exponents γ in the degree distribution, $P(k) \sim k^{-\gamma}$. In Fig. 3(a) we observe that for $\beta = 0$ (no adaptation) all the congestion thresholds differ and are very low, $p/\mu \sim 0$, with the SF network being more vulnerable to congestion as γ decreases. Second, we introduce the adaptation mechanism in the system with $\beta = 0.01$ and from Fig. 3(b) we observe that all the critical points are shifted to the maximum value $p/\mu \sim 1$. Finally, we extract the (highly reciprocal) weight distribution obtained at the critical point and we store it as a static weighted network for traffic routing and run the traffic process with no adaptation. In Fig. 3(c) we show that the critical point for all the networks is still close to the maximum. Thus, the reciprocal networks obtained from adaptation are able to keep the systems free of congestion up to $p/\mu \simeq 1$, thus maximizing the traffic capacity of the networks.

In Fig. 3(d), we show the effects that adaptation causes on link weights as the load of traffic increases. From the plots we observe that network hubs, placed in the center of the networks, need to decrease the weight of their connections progressively in order to avoid congestion. In this way, in the optimal regime, $p/\mu = 1$, almost all the links connected to a hub will have lower values than in the original system. Conversely, due to reciprocity, those nodes with lower degree (placed in the periphery of the networks) distribute the excess weight (remaining after the weakening of their connections with the hubs) by reinforcing their connection with their neighbors. Thus, as p/μ increases, the links associated with low-degree nodes increase their value, allowing a better distribution of the traffic and letting the system reach optimality.

Finally, we consider a more realistic scenario for the traffic dynamics. Until now, packets performed limited paths of average length $1/\mu$ and then disappear from the network. In the new realistic scenario, packets are born at a constant rate p at each node of the network. These new packets are assigned a particular destination node (randomly chosen among the $N - 1$ remaining nodes). In this way, the packets move across the network until their corresponding target nodes are reached.

We study the above realistic routing model by monitoring the congestion diagram $\rho(p)$ when no adaptation ($\beta = 0$) takes place and when the adaptive dynamics works with $\beta = 1$. The results are reported in Figs. 4(a) and 4(b) for ER and SF networks, respectively. It is clear that, in the absence of adaptivity, SF networks become congested much earlier than

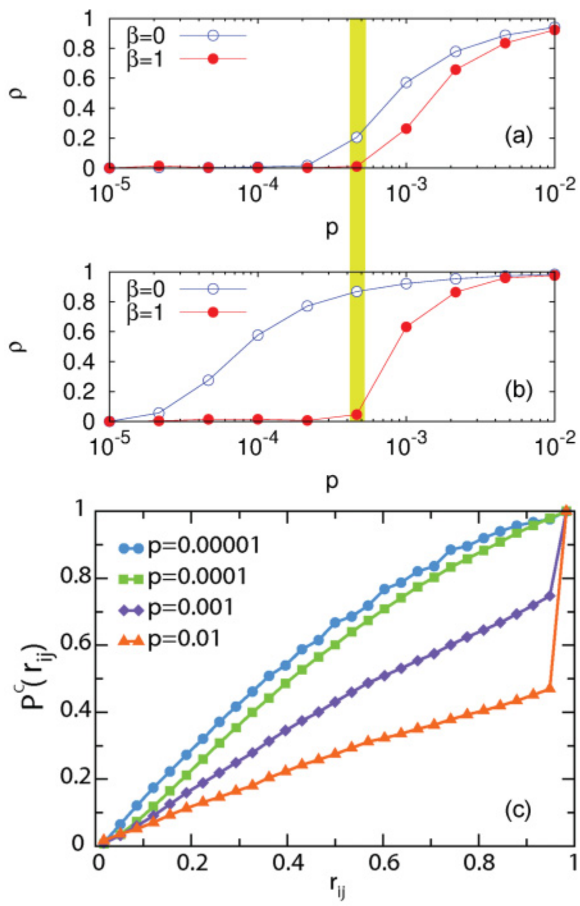

FIG. 4. (Color online) Panels (a) and (b) show the congestion diagram $\rho(p)$ in the traffic model with prescribed packets' destinations for ER and SF networks with 1000 nodes and an average degree of $< k >= 6$ (The SF has an exponent $\gamma = 3$). In both panels we show the diagrams corresponding to $\beta = 0$ (no adaptation) and $\beta = 1$ (strong adaptation). Panel (c) shows the cumulative distribution for the reciprocity of links, $P^c(r_{ij})$.

ER graphs (as previously observed in Fig. 1). However, as it was also the case for the simplified routing model, the capacity of both networks is enhanced up to the same critical point. Thus, by dropping parameter μ to incorporate realistic conditions into the traffic dynamics, the adaptive network is able to reproduce the optimization of network capacity regardless of the heterogeneity of backbone architecture. Moreover, in Fig. 4(c) we show the evolution of the cumulative distribution for the reciprocity of links $P^c(r_{ij})$ as the traffic load (p) increases in SF networks. We observe that, as p increases approaching p_c, adaptivity progressively shapes the weight topology into a highly reciprocal one, thus recovering the behavior observed in the simplified routing model [see Fig. 2(b)].

IV. CONCLUSIONS

Summing up, we have constructed a model of adaptive networks aimed at balancing the weight distribution of a communication network as a function of local traffic conditions. We have shown that, regardless of the heterogeneity pattern of the network backbone, it is possible to achieve optimal behavior, i.e., to delay the congestion point up to its maximum possible value, thus obtaining weighted networks with maximal traffic capacity. We have analyzed the microscopic details of the configurations obtained, observing that link reciprocity is the selected mechanism to achieve such optimal networks. These results have been tested in both a simplified routing model, assuming a constant rate for the death of the packets, and a more realistic scenario, in which packets are assigned a destination node so they remain in the network until they reach their corresponding targets.

Furthermore, we have shown that when using the reciprocal configurations obtained from adaptivity in a static way it is also possible to achieve optimal behavior. Therefore, adaptivity can also be used as an efficient methodology for the design of communication networks. In this way, a given static backbone of interactions can be assigned a reciprocal weight distribution that allows to operate in an optimal way. This last result links with the recently observed [37] reciprocal patterns observed in the axonal pathways of mammalian brain networks. Thus, as natural selection in the brain connectome, the adaptive process introduced here overcomes the constrains imposed by the structure of the backbone of interactions by selecting reciprocal wirings for the communication channels.

ACKNOWLEDGMENTS

This work has been supported by the Spanish DGICYT under projects FIS2008-01240, FIS2009-13364-C02-01, FIS2009-13730-C02-02, FIS2010-18639, FIS2011–25167, and MTM2009-13848; by the Generalitat de Catalunya and the Comunidad de Aragón through Project No. 2009SGR0838, FMI22/10; and by the J.S. McDonnell Foundation Research Award. J.G.G. is supported by MICINN through the Ramón y Cajal program.

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