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**Regular** Article

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**Abstract.** Congestion in transport networks is a topic of theoretical interest and practical importance. In this paper we study the flow of vehicles in urban street networks. In particular, we use a cellular automata model on a complex network to simulate the motion of vehicles along streets, coupled with a congestion-aware routing at street crossings. Such routing makes use of the knowledge of agents about traffic in nearby roads and allows the vehicles to dynamically update the routes towards their destinations. By implementing the model in real urban street patterns of various cities, we show that it is possible to achieve a global traffic optimization based on local agent decisions.

## 1 Introduction

Traffic optimization has always been a crucial issue in the context of communication and transportation systems [1-5]. A transport network is a network of roads, streets, pipes, power lines, or nearly any structure which permits either vehicular movement or the flow of some commodity. In most of the developed countries, transportation infrastructures originally designed to carry a defined amount of traffic are often congested by an overwhelming request of resources: this is the case of railroads, airplane connections and, of course, urban streets. A naive solution to the problem consists in expanding the infrastructure to match the increasing demand. However, this is not always possible due to limitations in available space, negative outcomes or shortness of resources. A better approach is to carefully tune the behavior of the existing infrastructures to efficiently exploit their actual structure and accommodate the new traffic demands.

The study of congestion in complex networks has mainly focused on information systems, such as gridcomputing networks or the Internet [4,6]. In such context, diverse solutions have been proposed in order to increase the network load avoiding the onset of congestion [7]. In particular, congestion-aware routing, in which the nodes of the network (the routers) redirect dynamically the information packets across the less congested paths, has proved to improve notably the capacity

of the network [8,9]. Particularly, it has been shown how a limited knowledge of local network structure can largely improve routing and navigability [10,11]. It thus seems possible to make use of similar kinds of routing strategies in transport networks. On the other hand, transport networks present important features different from information systems. First, in a transport network the links (i.e. the roads) carry the flow of vehicles, whereas the nodes are just intersections between links. Therefore, one cannot neglect the dynamics that occurs along the links: the quality of vehicle movement along the roads characterizes the functioning of transport systems. Another important feature is that, since transport networks are embedded in the real space, congestion is not located at particular nodes of the system (such as the hubs in information systems) [12] but it geographically spreads across the network from bottlenecks and it may eventually affect a large portion of the system. Therefore, to study congestion-aware routing of vehicular traffic in transport networks it is necessary to incorporate the above two ingredients.

In this paper, we focus on the realistic scenario in which only a *local knowledge* of congestion is available and used by the agents to modify their routes. Our model is implemented in terms of vehicular traffic and we assume that drivers know the shortest paths to their destinations and, simultaneously, they are aware about the congestion of nearby roads. Both informations are easily accesible nowadays from navigating systems, visual inspection and shortrange wireless communication with other vehicles [13]. We

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**Table 1.** City networks considered: N and K are the number of nodes and links in the network, W and  $\langle l \rangle$  are respectively the total length of the edges and the average edge length (both in meters) [14]. We report the value  $\alpha^*$  that maximizes the average number of completed routes per vehicle (see text).

City	N	K	W	$\langle l \rangle$	$\alpha^*$	$lpha^*$
					(L = 0.06)	(L = 0.1)
Barcelona	210	323	36179	112.01	0.63	2.04
Bologna	541	773	51219	66.26	0.87	1.54
Brasilia	179	230	30910	134.39	1.52	2.09
Los Angeles	240	340	38716	113.87	1.82	2.43
London	488	730	52800	72.33	1.02	1.26
New Delhi	252	334	32281	96.56	1.49	2.21
New York	248	419	36172	86.33	0.74	1.77
Washington	192	303	36342	119.94	1.33	2.14

show how, by conveniently controlling agent decisions, it is possible to minimize the overall congestion of the system and achieve performances at least as good as those obtained with complete (global) traffic information.

## 2 The model

The three ingredients of the model are: (i) the substrate graph, (ii) the vehicular dynamics along the streets, (iii) the routing at street crossings.

## 2.1 Urban graphs

The dynamics of vehicles takes place on top of urban graphs. We consider the street patterns of a city as a weighted graph with N nodes and K edges. Each edge of the network represents a street, along which vehicles move. whereas nodes account of intersections between streets. The weight of each edge is proportional to the length of the road [14]. Here we assume for simplicity that each edge allows movement of vehicles in both directions. We have considered eight networks representing 1-square mile samples of urban street patterns of real cities [14] (see Tab. 1 for details). The above city set ranges from *self*organized cities, such as Bologna, Barcelona and London, grown through a continuous process out of the control of any central agency, to *grid-like* cities such as Los Angeles, New Delhi and Washington, realized over a short period of time as the result of urban plans, and usually exhibiting grid structures.

#### 2.2 Model of vehicular dynamics

The vehicular dynamics along the links of the urban graph is simulated by means of cellular automata [15–17]. In particular we use the *Nagel-Schreckenberg* (NaSch) model [18]. To this end, every link (street) of the city graph is divided into a sequence of cells of equal length (5 m) so that no more than one vehicle can occupy a cell at every time step. Each vehicle is assigned a velocity of v

cells per time step [19] in the range  $v \in [0, v_{max}]$ . We set  $v_{max} = 3$ , corresponding to the typical maximum velocity of about 50 Km/h inside urban areas. According to the NaSch model, vehicles accelerate (decelerate) when the next cells are empty (occupied). Additionally, the intersections between streets (the nodes of the graph) also allow only one vehicle at a given time, so that several vehicles coming from different adjacent streets may compete for the same intersection [20]. For this reason, vehicles experience a slow down while approaching a road intersection, as if the end of the lane presented a hindrance, so that they arrive at the last cell of the edge with v = 0. At this point, a vehicle waits to enter into the node [21], where it gets stuck until the first cell of the edge in the proper outgoing direction is free. This waiting locks the incoming flows from the edges. Therefore, bottlenecks are created from the nodes and spread along the edges (roads) of the graph.

#### 2.3 Routing strategy

How vehicles decide their outgoing direction when leaving a node? Here we implement a *congestion-aware* routing as a minimization problem that takes into account the length of the path and also the traffic along the outgoing edges. When a vehicle is at a node *i* it needs to choose a new node *n* in its neighborhood  $\Gamma_i$  as the next hop on its path towards the destination *t*. For each of the neighbouring nodes *n*, a *penalty function*  $P_n$  is defined as:

$$P_n = (d_{in} + d_{nt})(1 + c_{in})^{\alpha}, \quad n \in \Gamma_i$$
(1)

where  $d_{in}$  is the distance between nodes i and n, and  $c_{in}$ ( $c_{in} \in [0, 1]$ ) represents the congestion of the link  $i \to n$ , measured as the fraction of occupied cells in the link. The exponent  $\alpha \ge 0$  accounts for the weight given to the local congestion in the drivers decision. The vehicle chooses the node n with the minimum penalty  $P_n$ . If  $c_{in} = 0$  the penalty function  $P_n$  is nothing else than the length of the shortest path to t, passing by node n. When  $c_{in} \ge 0$  the entire shortest path length is corrected by the factor  $(1 + c_{in})^{\alpha}$ . In this way, we assume that the vehicle projects the congestion  $c_{in}$  of the link  $i \to n$  on the entire path



Fig. 1. Urban street network of Bologna (one-square-mile sample). Links are drawn with a thickness proportional to their congestion c: from (a) to (c) a more congestion-aware strategy rules the same amount of traffic, which progressively flows in a larger number of streets. The traffic load is fixed at L = 0.2, while the values of  $\alpha$  considered are respectively equal to 0, 1 and 2.



Fig. 2. Urban street network of London (one-square-mile sample). Links are drawn with a thickness proportional to their congestion c: from (a) to (c) a more congestion-aware strategy rules the same amount of traffic, which progressively flows in a larger number of streets. The traffic load is fixed at L = 0.2, while the values of  $\alpha$  considered are respectively equal to 0, 1 and 2.

 $i \to n \to t$ . Note that, when  $\alpha = 0$ , local congestion plays no role in the routing and vehicles follow the shortest paths to their respective destinations.

## **3 Results**

We evaluate our model through computer simulations. First, we investigate how local knowledge can be exploited by a congestion-aware routing strategy; then, we report how the availability of global knowledge about congestion affects the traffic optimization.

We initially place a number of vehicles proportional to the number of cells in the network, so that the network load,  $L = \frac{V}{C}$  (i.e. the ratio between the number of vehicles V in the network and the total number of cells C), is the same for all the cities considered. Initially, the vehicles are assigned a random source (their initial location) and a random destination node. At each time step, vehicles move (if possible) in the system according to the NaSch rules and the congestion-aware routing, equation (1). Finally, when a vehicle reaches its destination it is randomly assigned to a new destination node, so that L is constant in time. After an initial transient dynamics, the system reaches a steady state in which data is collected.

#### 3.1 Local knowledge

We report the dynamical behavior of the model in the different cities considered as a function of the routing,  $\alpha$ , and the network load, L. First, in Figures 1 and 2 we show the congestion pattern across the streets of Bologna and London for a load L = 0.2, and for three different values  $\alpha = 0, 1, 2$ . It is clear that the larger the value of  $\alpha$ , the more homogeneously distributed is the traffic. We have found similar results in all the cities under analysis. In order to study and quantify the effects of congestion in the vehicular motion we analyze the so-called *fundamental diagram* [18]. The fundamental diagram represents the network mean flux f (the average value of fluxes on streets)

as a function of the traffic load L. This is shown in Figure 3a for different values of  $\alpha$ . Each of the curves f(L)exhibits the two typical traffic phases: *free-flow* (in which the flow increases as a function of the load) at low L, and congested-flow (the flow decreases as a function of the load) at higher values of L. Interestingly, both the value of maximum flow and the transition point from the free-flow regime to the congested-flow regime increase with  $\alpha$ .

In particular, when the vehicles follow shortest paths  $(\alpha = 0)$ , they tend to concentrate in nodes and link with high betweenness, thus, even a small density of vehicles can result in a heavy load at high betweenness streets. This gives rise to the formation of clusters of jammed vehicles that cannot easily move and, although large regions of the city are quite uncongested as shown in Figures 1a and 2a, the overall flux of the network is largely reduced. We observe similar behavior in all the other cities under analysis.

The above result is confirmed by the sharp decrease of the average vehicle speed v as a function of L for  $\alpha = 0$ shown in Figure 3b. On the other hand, as the routing strategy becomes more congestion-aware, the traffic is diverted from shortest path to free (and longer) paths causing that both the mean speed of the vehicles and the mean flux on the streets are largely increased with respect to the case  $\alpha = 0$ , since more vehicles are now able to reach their destinations without being blocked in congested nodes. More interestingly, we observe in Figure 3b that for  $\alpha = 2$ and 3 the average velocity v does not decrease monotonically as a function of L but it reaches a maximum  $v_{max}(\alpha)$ at some load  $L^*(\alpha)$ . However, having a larger average velocity does not imply that the network is working in a more efficient way. In fact, for large values of  $\alpha$ , the vehicles can move faster running across longer paths at the expense of delaying the arrival to their destinations.

A measure of the efficiency of the routing is the average number of routes r completed by a vehicle during one hour. The value of r is reported in Figure 3c as a function of L. The results indicate that, because of congestion, the number of completed routes decreases when the number of vehicles in the city increases. The precise dependence of r on the load is related to the value of  $\alpha$ . For  $\alpha = 0$ , the average vehicle speed has a very sharp drop as the load increases. For the congestion-aware strategy with  $\alpha = 1$ the decrease is smoother than for  $\alpha = 0$ , while for  $\alpha = 2$  $(\alpha = 3)$  the value of r is smaller than that of  $\alpha = 1$  when  $L < 0.1 \ (L < 0.14)$ , but larger when  $L > 0.1 \ (L > 0.14)$ . In practice, for a given load L, the function  $r(\alpha)$  shows a maximum at some  $\alpha^*$  (see inset in Fig. 3c). The value of  $\alpha^*$ is seen to increase with the load L, pointing out that the more congested the network is, the more congestion-aware has to be the routing to reach the optimal functioning. On the other hand, the maximum value of  $r, r(\alpha^*)$ , decreases with L. Similar results as those shown for Bologna have been found for the other cities studied. The best routing exponents  $\alpha^*$  obtained for two realistic values of the vehicle density, namely L = 0.06 and L = 0.1 [3], are reported in Table 1. It is clear that the optimal value  $\alpha^*$  depends strongly on the particular topology of the urban graph,



Fig. 3. (Color online) Average street flow (a), average vehicle speed (b), and average number of completed routes per vehicle per hour (c) as a function of the network load L. The network considered is that of the city of Bologna.

since cities with similar number of nodes, links and average link length exhibit different values. Particularly, we note that self-organized cities such as Bologna, Barcelona and London exhibit smaller values of  $\alpha^*$  both for L = 0.06 and L = 0.1 with respect to other cities, whereas grid-like cities as Los Angeles, New Delhi and Washington are characterized by the largest values of  $\alpha^*$ . The city of New York does not fit well in this classification, mainly because its network is heavily affected by the shape of the island of Manhattan.



Fig. 4. (Color online) Ratio between the average edge betweenness of the road where the vehicles are moving and the average edge betweenness of the city as a function of the network load L and for different values of the parameter  $\alpha$ . The network considered is that of the city of Bologna.

We report in Figure 4 the ratio b between the average edge betweenness of the roads where the vehicles are moving and the average edge betweenness of the whole network, for different values of L and  $\alpha$ : when this value is higher than 1, vehicles are moving in streets with high betweenness, while if this value approaches 1 vehicles are more evenly distributed across the network. We observe that when  $\alpha = 0$  vehicles are more likely to use streets with high betweenness while, on the contrary, when  $\alpha > 0$ is adopted the average betwenneess of the used roads is lower, since vehicles spread also in less central routes in order to avoid congestion in central areas. Finally, when the load is high the vehicles are moving so slowly that they occupy, on average, the whole city and the differences between the various strategies disappear. This analysis shows that a more congestion-aware routing strategy diverts vehicles on roads with lower betweenness.

#### 3.2 Global knowledge

Even though a precise information on global congestion is in practice rarely available, we finally study the case in which each vehicle knows exactly the congestion in every link of the network. Namely, we compare the results obtained with equation (1) with those obtained with the penalty function

$$P_n = (d_{in} + d_{nt})(1 + \langle c_{int} \rangle)^{\alpha}, \qquad (2)$$

where  $\langle c_{int} \rangle$  accounts of the average road congestion along the path from *i* to *t* passing by *n*. Therefore, we now project the effect of the average congestion of the path (obtained from the global knowledge) on the path distance.

The average number r of completed routes per hour is reported in Figure 5. The figure shows that the routing with global knowledge does not perform much better than that of equation (1). Additionally, at large loads, local knowledge outperforms global knowledge in terms of r. Moreover, when global information is taken into account, the optimal routing  $\alpha^*$  increases. This is related to the



Fig. 5. (Color online) Average number of completed routes per vehicle for the global-aware strategy. In the insets we report r as function  $\alpha$ . The network considered is that of the city of Bologna.

fact that, since congestion in links close to vehicle location is always up-to-date and therefore accurate, routing based on local congestion needs lower values of  $\alpha^*$  to divert vehicles on free streets.

## 4 Conclusions

Congestion in transportation and communication networks is a serious problem for both public goods and users time. In this paper we have integrated the three essential ingredients of vehicular traffic in urban settings, namely the graph structure of urban patterns, a cellular automata model for vehicular dynamics along the links and the use of a congestion-aware routing inspired to data traffic in the Internet. We have provided a simple and feasible model where only local information about traffic congestion is used to route vehicles. Our results show how each individual agent can better organize its motion based on its limited local knowledge of traffic and, at the same time, achieve optimal performances at the global system level. We have implemented this model in several real cities with different structural properties showing how the optimal vehicle routing strategy depends on the network topology. Finally, we have shown that, counterintuitively, a (unfeasible) routing based on a global knowledge of congestion is not suited when the load of vehicles is large. The proposed routing model is general enough to be applied to several types of human transport networks. Moreover, the dynamic and distributed nature of the model allows several applications to be built which may implement the congestion-aware strategy to improve the available vehicle's navigation systems.

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