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Amplitude dynamics favors synchronization in complex networks

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In this paper we study phase synchronization in random complex networks of coupled periodic oscillators. In particular, we show that, when amplitude dynamics is not negligible, phase synchronization may be enhanced. To illustrate this, we compare the behavior of heterogeneous units with both amplitude and phase dynamics and pure (Kuramoto) phase oscillators. We find that in small network motifs the behavior crucially depends on the topology and on the node frequency distribution. Surprisingly, the microscopic structures for which the amplitude dynamics improves synchronization are those that are statistically more abundant in random complex networks. Thus, amplitude dynamics leads to a general lowering of the synchronization threshold in arbitrary random topologies. Finally, we show that this synchronization enhancement is generic of oscillators close to Hopf bifurcations. To this aim we consider coupled FitzHugh-Nagumo units modeling neuron dynamics.

Synchronization of interacting units is a universal collective behavior appearing in a variety of natural and artificial systems[1,2]. The most used mathematical formalisms to study how synchronization shows up, such as the paradigmatic Kuramoto model[3], consider each dynamical unit as a pure phase oscillator. This coarse-grained dynamical description lies in the following assumption: since each isolated unit has a stable limit cycle oscillation at a given *natural* frequency, it is enough to study the variable accounting for the motion along this limit cycle (the phase), while the other dynamical variables may be ignored. This approach is justified provided the attraction to the limit cycle is strong compared to the coupling between dynamical units[4]. On the contrary, if the coupling is strong or the attraction to the limit cycle is weak, different phenomena, whose analysis demands both amplitude and phase dynamics, such as oscillation death[5,6] and remote synchronization[7–9], can occur.

The Stuart-Landau (SL) model[10,11] allows to bridge the gap between the simplicity of the Kuramoto model and the completeness of the amplitude-phase frameworks. In particular the SL model may describe, by means of a single parameter, both the behavior close to the Hopf bifurcation, which represents an instance where the attraction to the limit cycle is weak, and the case where, on the contrary, the motion is purely confined to the limit cycle thus behaving as a phase oscillator.

Synchronization of pure phase oscillators interacting according to a complex network topology has been widely studied during the last decade[12]. These studies have covered different topics such as the conditions for the onset of synchronization[13], the path towards it[14], the interplay between topology and dynamics[15,16], the emergence of first-order transition[17–20], the effect of noise on the robustness of the global state[21], among others. However, interesting issues about the interplay of a network topology of interactions and the coupled dynamics of the amplitude and phase of the oscillators are often ignored from scratch, due to the underlying dynamical framework chosen.

In this paper, we adopt the framework of SL oscillators coupled according to different network configurations and compare with its reduction to pure phase oscillators. Our goal is to investigate the effect of the amplitude dynamics near the Hopf bifurcation on the synchronization in random complex networks. The most striking result is that, when the amplitude dynamics is not negligible and the natural oscillation frequencies of the nodes are not homogeneous, synchronization is enhanced regardless of the topology of the underlying network. The result holds for oscillators close to the Hopf bifurcation and in particular we also discuss its implications for neuron-like dynamics, by illustrating the behavior for the FitzHugh-Nagumo model.

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Author Contributions

L.V.G., J.G.-G. and M.F. conceived the research, L.V.G. carried out the simulations. All authors analyzed the data and wrote the manuscript.

Additional Information

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