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# Network bipartivity and the transportation efficiency of European passenger airlines

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## HIGHLIGHTS

- A new way for calculating the bipartivity of networks is introduced.
- The bipartivity of the European Low Cost Carriers and Traditional airlines shows significant differences.
- Alliances and airline mergers decrease the bipartivity of the corresponding networks.
- Bipartivity is strongly correlated with the transportation efficiency of the European airlines.

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## ABSTRACT

The analysis of the structural organization of the interaction network of a complex system is central to understand its functioning. Here, we focus on the analysis of the bipartivity of graphs. We first introduce a mathematical approach to quantify bipartivity and show its implementation in general and random graphs. Then, we tackle the analysis of the transportation networks of European airlines from the point of view of their bipartivity and observe significant differences between traditional and low cost carriers. Bipartivity shows also that alliances and major mergers of traditional airlines provide a way to reduce bipartivity which, in its turn, is closely related to an increase of the transportation efficiency.

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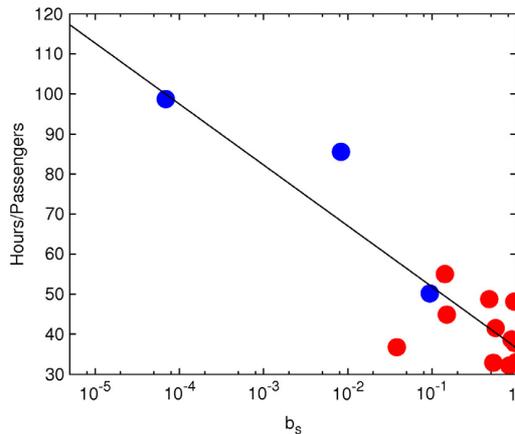
## 1. Introduction

A fundamental characteristic of complex systems is that, in general, they are networked. Thus, complex networks, which represent the skeleton of such complex systems, are ubiquitous in many real-world scenarios, ranging from the biomolecular – those representing gene transcription, protein interactions, and metabolic reactions – to the social and infrastructural organization of modern society [1]. Mathematically speaking, these networks are graphs with the nodes representing the entities of the system and the edges representing the “relations” among those entities. From a structuralist point of view of nature it could be claimed that a large proportion of the properties of these complex systems is determined by the structure of these networks. The question about what do we mean by “the structure” of these networks is a tricky

one. This situation is reminiscent of Edgar Allan Poe’s response about what is the structure of a strange ship: “*What she is not, I can easily perceive—what she is I fear it is impossible to say*” [2]. Then, the pragmatic approach used in network theory and beyond is to consider structural invariants which characterize some portions of this structure which in global terms scape to our formal definitions. That is the reason why we have such a large amount of structural invariants, i.e., numbers that characterize some properties of the network independently of the labeling of nodes and edges [3]. Such invariants include the average path length, clustering coefficients, densities, assortativity coefficients, and many more (see [1,3] for non-exhaustive lists).

The concept of network bipartivity is one that has given rise to some structural invariants to characterize how much bipartivity a network has. Bipartivity has long been studied in graph theory as a black-and-white concept. That is, just by considering that a graph is or is not bipartite. However, in the noughties there were three papers that attempted to characterize how much bipartivity a non-bipartite graph has. The pioneering work of Holme et al. proposed the first of such measures in 2003 by using computational

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**Fig. 5.** Illustration of the relation between the airline carriers efficiency and the spectral bipartivity index. The airline efficiency is measured by the number of passengers transported divided by the number of hours flown. Blue nodes correspond to low-cost companies and red nodes to traditional ones. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Motivated by the above findings we have observed that traffic efficiency (as revealed from real data about the traffic flow of each airline) is strongly and negatively correlated with the bipartivity of its network. Therefore, bipartivity provides a good descriptor of the efficiency of transportation networks and can be used to test the goodness of alliances and possible mergers of airlines. The general way of computing bipartivity may allow its use to evaluate more complex mixing of transportation networks such as the sharecodes of flights between different airlines, which is the usual collaboration benchmark in air transportation systems.

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