

## Synchronization unveils the organization of ecological networks with positive and negative interactions

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Network science has helped to understand the organization principles of the interactions among the constituents of large complex systems. However, recently, the high resolution of the data sets collected has allowed to capture the different types of interactions coexisting within the same system. A particularly important example is that of systems with positive and negative interactions, a usual feature appearing in social, neural, and ecological systems. The interplay of links of opposite sign presents natural difficulties for generalizing typical concepts and tools applied to unsigned networks and, moreover, poses some questions intrinsic to the signed nature of the network, such as how are negative interactions balanced by positive ones so to allow the coexistence and survival of competitors/foes within the same system? Here, we show that synchronization phenomenon is an ideal benchmark for uncovering such balance and, as a byproduct, to assess which nodes play a critical role in the overall organization of the system. We illustrate our findings with the analysis of synthetic and real ecological networks in which facilitation and competitive interactions coexist. *Published by AIP Publishing.*  
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**In the last decade, network science has provided the ideal benchmark to encode, analyze, and understand the complex relationships that are established in large scale systems of nature as disparate as the Internet or the brain. As databases become more abundant and complete, we face the challenge of analyzing networks containing multiple types of connections. Here, we tackle the case of signed networks, where the interactions can be either positive or negative. On one hand, we illustrate how synchronization processes capture the organization of this kind of graph into a set of modules interacting negatively among them. With this in mind, we apply the synchronization benchmark to real systems. In particular, we characterize the ecological balance between facilitation and competition in plant communities and analyze the role of species in their organization.**

### I. INTRODUCTION

Synchronization is perhaps the most paradigmatic example of collective behavior as it is recurrently found at different levels of complexity.<sup>1-3</sup> In fact, the emergence of spontaneous synchronization in systems of coupled dynamical units is at the core of many coordinated tasks, from cognitive processes in the brain to the unfolding of collective behaviors in social systems.<sup>4-6</sup> In the last decade, network theory has unveiled that the topology of the interactions in a complex system has important effects on the development of

collective behaviors.<sup>5,7</sup> Following this direction, the study of synchronization in networks has attracted a lot of attention<sup>8</sup> to shed light on the role that the network structure plays on the emergence of synchronized states.<sup>9-17</sup>

A typical setting of the former works consists of associating a dynamical system to each node, whereas the couplings between pairs of dynamical units are mediated by the links of the network. However, other studies have also covered adaptive networks<sup>18</sup> whose structure is shaped by the microscopic synchronization patterns<sup>19-22</sup> or systems of mobile oscillators moving in a continuous space.<sup>23-27</sup> With relatively few exceptions,<sup>28-36</sup> the hypothesis of these works is that the interactions between units are positive, so that the existence of a link between two coupled dynamical systems implies that they are prone to synchronize as the interaction between them is increased.

In this work, we address the scenario in which positive and negative interactions between nodes coexist in the same network. In this way, the increase of the interaction strength causes the attraction among those dynamical units interacting via positive links and a repulsive effect between those connected through negative edges. As shown recently by Anderson and co-workers,<sup>36</sup> the stable equilibrium reached consists of a dynamical partition of the network in which nodes sharing positive connections synchronize together while they avoid being dynamically close to those with whom negative interactions are at work. Here, our goal is to use this partition to extract information about the

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