Disorder Induced Control of Discrete Soliton Ratchets

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We demonstrate that directed transport of topological solitons in damped, biharmonically driven Frenkel-Kontorova chains can be strongly enhanced by introducing suitable *phase disorder* into the asymmetric periodic driving. From a collective coordinate formalism, we theoretically deduce an effective deterministic equation of motion governing the dynamics of the soliton center-of-mass for which we predict the dependence of maximal soliton drift on disorder strength according to recently proposed general scaling laws concerning directed transport induced by symmetry breaking of temporal forces. We find that these results are in excellent agreement with those of computer simulations of the original Frenkel-Kontorova chains.

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Directed transport without any net external force (the so-called ratchet effect [1–4]) has today become a fundamental field of multidisciplinary research in nonlinear science. This is mainly because the phenomenon has an extremely broad range of possible technological applications, e.g., in molecular switches, rectifiers, transistors, particle separation devices, and pumps, as well as owing to its great interest in biology where ratchet mechanisms are found to underlie the working principles of molecular motors [5]. Generally, directed transport induced by the ratchet effect has of now been understood as a result of the interplay among symmetry breaking, nonlinearity, and nonequilibrium fluctuations, where such fluctuations refer to temporal noise in most cases. Recently, in an effort to approach real-world ratchets, the effects of different kinds of spatial disorder on the transport properties of diverse model systems have been investigated [6–10]. Some unexpected disorder-induced phenomena were encountered, such as strong diffusive motion [8] and an enhancement of the transverse-rectified velocity of a driven fluid [10]. Thus, the role of spatial disorder on the dynamics of otherwise deterministic ratchets could shed some light on diverse experimental situations including particle separation techniques, dielectrophoretic trapping, and surface electromigration. Similarly, one would expect that studies of model systems of extended ratchets subjected to disordered driving forces could help to understand various transport processes in *heterogeneously* driven systems such as populations of biological motors. However, to the best of our knowledge, the transport effects of this type of quenched *temporal* disorder have yet to be discussed.

In the present Letter, we explore the use of phase disorder in external forces as a means of controlling directed transport generated by the ratchet effect. To be specific, we study this new effect of disorder in the context of an example of discrete soliton ratchets induced by symmetry breaking of temporal forces [11–15]. In particular, we study arrays of damped, driven Frenkel-Kontorova (FK) chains $ml^2\ddot{\theta}_n + mgl\sin\theta_n = -\gamma\dot{\theta}_n + F_n(t;\varphi_n) + \kappa(\theta_{n+1} + \theta_{n-1} - 2\theta_n)$, $F_n(t;\varphi_n) \equiv \tau\eta\sin(\omega t) + \tau\sin(2\omega t + \Phi) - \tau\eta\sin(2\omega t + \Phi + \varphi_n)$ [16] using, for convenience, the dimensionless form

$$\begin{aligned}\frac{d^2u_n}{dt'^2} = &-\frac{K}{2\pi}\sin(2\pi u_n) - \alpha\frac{du_n}{dt'}\\ &+ \Gamma[\eta\sin(\Omega t') + \sin(2\Omega t' + \Phi)\\ &- \eta\sin(2\Omega t' + \Phi + \varphi_n)] + u_{n+1} - 2u_n + u_{n-1},\end{aligned} \tag{1}$$

where $u_n \equiv \theta_n/(2\pi)$ is the phase of the nth pendulum, $\alpha \equiv \gamma/\sqrt{ml^2\kappa}$ is the damping coefficient, Φ is a homogeneous initial phase, $K \equiv lmg/\kappa$ measures the strength of the substrate potential, $\Gamma \equiv \tau/(2\pi\kappa)$ and $\eta \in [0, 1]$ are amplitude factors, and $t' \equiv t\sqrt{\kappa/(ml^2)}$, $\Omega \equiv \omega\sqrt{ml^2/\kappa}$ are dimensionless time and frequency, respectively. As is well known, many physical and biological phenomena including ladder networks of discrete Josephson junctions, adsorbed monolayers, DNA dynamics, and charge density wave conductors can be investigated in the framework of the FK model [17]. We consider a finite chain of N particles with the following periodic boundary condition: $u_0 = u_N - N - 1$, $u_{N+1} = u_1 + N + 1$ to keep the analysis close to experimental possibilities (e.g., an ac-biased annular array of Josephson junctions). We assume that in the absence of disorder (i.e., when all driving forces have the same initial phases $\varphi_n = 0$) the deterministic biharmonic force $F(t;\varphi_n = 0) \equiv \tau\eta\sin(\omega t) + \tau(1-\eta)\sin(2\omega t + \Phi)$ yields directed kink transport (DKT) whose strength depends upon the particular values of the remaining parameters, and study the disorder-induced effect on transport properties by randomly choosing the initial phases φ_n uniformly and independently from the intervals $[-k\pi, k\pi]$ with $k \in [0, 1]$ being the disorder parameter. As is well known, a collective coordinate formalism (CCF) [18] permits one to describe the motion of the kink center-of-mass, $X(t)$, by means of an effective ordinary differential equation, which is a perturbed pendulum for the FK model (see [19] and references therein). Thus, the application of CCF to Eq. (1) by assuming a sine-Gordon profile for the discrete soliton, $u_n = n \pm (2/\pi)\tan^{-1}\{\exp[n - X(t)]/l_0\}$, yields the family of randomly perturbed pendulum equations (one for each sampling of the uniform distributions)

$$\begin{aligned}\frac{d^2z}{dt^{*2}} = &-\sin z - \delta\frac{dz}{dt^*} + \Gamma^*[-\eta\sin(\tilde{\Omega}t^*) - \sin(2\tilde{\Omega}t^* + \Phi) \\ &+ \eta A(\varphi_n, k)\sin(2\tilde{\Omega}t^* + \Phi) + \eta B(\varphi_n, k) \\ &\times \cos(2\tilde{\Omega}t^* + \Phi)]\end{aligned} \tag{2}$$

where $z \equiv 2\pi X$, $t^* \equiv \Omega_{PN}t'$, $\delta \equiv \alpha\Omega_{PN}^{-1}$, $\Gamma^* \equiv \pi^3 l_0\Gamma\Omega_{PN}^{-2}$, and $\tilde{\Omega} \equiv \Omega\Omega_{PN}^{-1}$, where Ω_{PN} and l_0 are the Peierls-Nabarro frequency and the soliton width, respectively $\{\Omega_{PN}^2/(2\pi) = (\pi^3 + 2\pi^5 l_0^2)/[6l_0\sinh(\pi^2 l_0)]$, $l_0 \simeq 1/\sqrt{K}\}$. Here, $A(\cdot, k) \equiv \sum_n \cos(\cdot)\mathrm{sech}[(2n\pi - z)/(2\pi l_0)]$, $B(\cdot, k) \equiv \sum_n \sin(\cdot)\mathrm{sech}[(2n\pi - z)/(2\pi l_0)]$ are disorder-induced random amplitudes having averages $\langle A(\cdot, k)\rangle = \mathrm{sinc}(k\pi)$, $\langle B(\cdot, k)\rangle = 0$ and standard deviations $\sigma[A(\cdot, k)] = \sqrt{[1 + \mathrm{sinc}(2k\pi)]/2 - \mathrm{sinc}^2(k\pi)}$, $\sigma[B(\cdot, k)] = \sqrt{[1 - \mathrm{sinc}(2k\pi)]/2}$, respectively, and where $\mathrm{sinc}(x) \equiv \sin(x)/x$ (cardinal sine) while angular brackets indicate average over the respective random initial phase. Numerical experiments indicate that the statistical properties of the clouds of points accumulated in the $A(\varphi_n, k) - B(\varphi_n, k)$ plane after a large number of different samplings of uniform distributions are accurately described by the aforementioned statistical quantities for almost all $k \in [0, 1]$. This means that one can consider the *effective* deterministic equation

$$\begin{aligned}\frac{d^2z}{dt^{*2}} = &-\sin z - \delta\frac{dz}{dt^*} - \Gamma^*\eta\sin(\tilde{\Omega}t^*) \\ &- \Gamma^*[1 - \eta\,\mathrm{sinc}(k\pi)]\sin(2\tilde{\Omega}t^* + \Phi)\end{aligned} \tag{3}$$

to reliably characterize the averaged effect of disorder on DKT. In the absence of disorder $[k = 0, \lim_{k\to 0}\mathrm{sinc}(k\pi) = 1]$, we study the dependence of the average kink velocity $\langle v\rangle \equiv \langle\dot{X}(t)\rangle$ on η and Φ while keeping the remaining parameters constant. This average is solely temporal when dissipation is sufficiently large for there to be a single attractor associated with the DKT, while an additional average over the initial conditions is needed otherwise. An illustrative example is shown in Fig. 1, where recently conjectured general scaling laws [20] which are directly applicable to the effective pendulum (3) are accurately confirmed by numerical simulations of the FK chain (1). Notice that this *in turn* provides strong support for the effective model (3). Indeed, for Γ *sufficiently* small but still above the depinning threshold, one typically finds that $\langle v\rangle$ scales as $\eta^2(1 - \eta)\cos(\Phi)$ in the practical absence of dissipation ($\alpha \simeq 0$), and hence maximal strength of DKT is achieved at the values $\eta = 2/3$, $\Phi = \{0, \pi\}$ where the two values of Φ correspond to DKT in opposite directions (see Fig. 1, top panel, a). As dissipation increases ($\alpha > 0$), the dependence of $\langle v\rangle$ on η and Φ becomes piecewise [15], although, for sufficiently weak dissipation, the numerical data still reasonably fit the above scaling law with an additional dissipation phase:

$$\eta^2(1 - \eta)\cos(\Phi + \Theta_{\mathrm{diss}}), \tag{4}$$

with the properties $\Theta_{\mathrm{diss}}(\alpha = 0) = 0$, $\max_\alpha\Theta_{\mathrm{diss}}(\alpha) = \pi/2$ [cf. [20], see Fig. 1, top panel, (b)–(d)]. In particular, version d shows the situation where the dissipation is sufficiently large for Θ_{diss} to already have reached its maximum value. The dependence of $\langle v\rangle$ on η is shown in Fig. 1 (bottom panel) for two values of Φ. The DKT for $\Phi = \pi/2$ (version a) is stronger than that for $\Phi = \pi/4$ (version b) over the (almost) complete range of η according to the theoretical prediction (4). To study the effect of disorder ($k > 0$), we calculated the mean velocity of the kink on averaging over several different samplings of uniform distributions, $\langle\langle v\rangle\rangle$, which reduces to the aforementioned average kink velocity $\langle v\rangle$ in the absence of

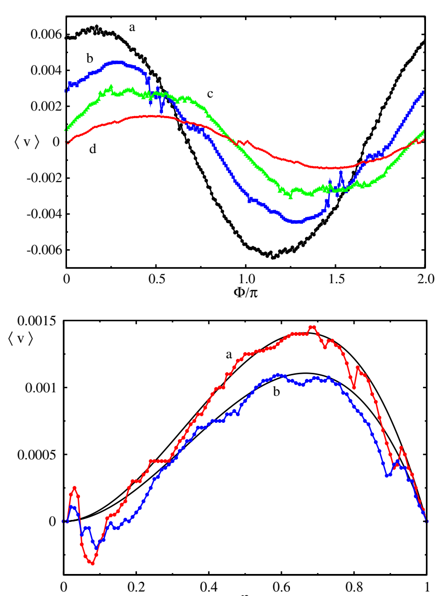

FIG. 1 (color online). Top panel. Dependence of the average kink velocity on the initial phase Φ for the FK chain [Eq. (1)] with $\eta = 2/3$ and four values of the dissipation coefficient: $\alpha = 0.001$ (a, near the Hamiltonian limit), $\alpha = 0.005$ (b), $\alpha = 0.01$ (c), and $\alpha = 0.0154161$ (d, near the maximum of the dissipation phase). Bottom panel. Dependence of the average kink velocity on the factor amplitude η for $\alpha = 0.0154161$ and two values of Φ: $\pi/2$ (a, i.e., $\Phi + \Theta_{\mathrm{diss}} \simeq \pi$) and $\pi/4$ (b), and where the solid lines represent the scaling given in Eq. (4). Fixed parameters: $N = 20$, $K = 0.5$, $\Gamma = 0.0022$, $\Omega = 0.002\pi$, $k = 0$ (absence of disorder).

disorder. We typically used 120 different samplings of the random initial phases to legitimate the comparison with the predictions from the effective Eq. (3). In this case, one expects [20] $\langle\langle v \rangle\rangle$ to scale as

$$\eta^2[1 - \eta \mathrm{sinc}(k\pi)] \cos(\Phi + \Theta_{\mathrm{diss}}); \qquad (5)$$

i.e., a disorder-induced *increase* of DKT should occur on average, as is indeed confirmed by numerical experiments. Figure 2 shows an illustrative example for several values of the disorder parameter. As was found for the purely deterministic case, one typically obtains strikingly good agreement between the numerical results and the predicted scaling [Eq. (5)]. Clearly, a *nonlinear* disorder-induced increase of the DKT strength occurs as the disorder strength is increased, i.e., $\langle\langle v \rangle\rangle_{\mathrm{max},k>0} \sim \langle v \rangle_{\mathrm{max},k=0} \mathrm{sinc}^{-2}(k\pi)$. Thus, this increase is maximal at optimal values of the initial phase Φ for fixed η [e.g., it is about 70% for $\Phi = \pi/2$, $k = 0.4$ see Fig. 2 (top panel)],

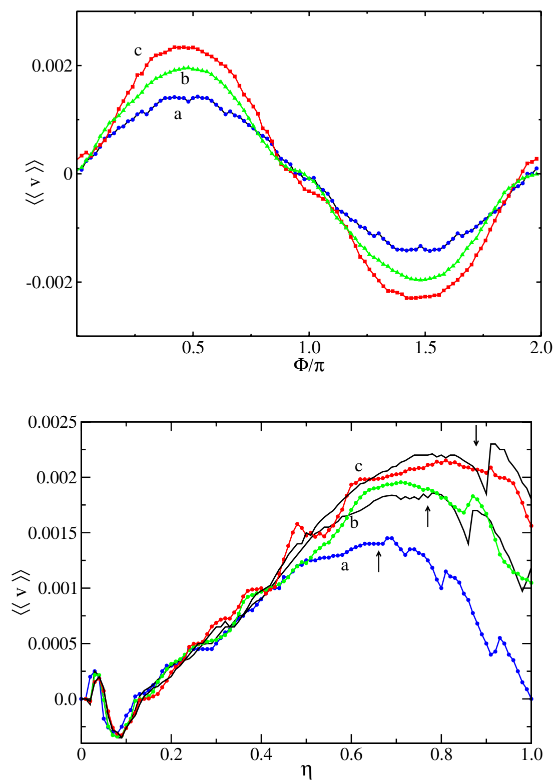

FIG. 2 (color online). Top panel. Dependence of the average kink velocity on the initial phase Φ for the FK chain [Eq. (1)] with $\alpha = 0.0154161$, $\eta = 0.7$ and three values of the disorder parameter: $k = 0$ (a), $k = 0.3$ (b), and $k = 0.4$ (c). Bottom panel. Dependence of the average kink velocity on the factor amplitude η for $\Phi = \pi/2$, $\alpha = 0.0154161$, and three values of the disorder parameter: $k = 0$ (a), $k = 0.3$ (b), and $k = 0.4$ (c). The results for the FK chain in the absence of disorder and with $\Gamma\eta$ scaled by its respective $\mathrm{sinc}(k\pi)$ factor [cf. Equation (5)] are indicated by solid lines. The three arrows indicate the positions of the respective maxima of η from Eq. (5). The fixed parameters are the same as in Fig. 1.

and at $\eta = \eta_{\max}(k = 0) \sin \mathrm{c}^{-1}(k\pi)$ for fixed Φ [see Fig. 2 (bottom panel)]. Also, we find that prediction (5) holds over a wide range of frequencies, while its accuracy as well as the strength of DKT decrease as $\Omega \to \Omega_{\mathrm{PN}}$ [i.e., as $\tilde{\Omega} \to 1$ cf. Equation (3)] as is shown in Figs. 3 and 4. This can be understood as follows. For sufficiently high $\tilde{\Omega}$, the kink dynamics (3) can be analyzed using the vibrational mechanics approach [21] by assuming that the $\tilde{\Omega}$-force is "slow" while the $2\tilde{\Omega}$-force is "fast." Thus, one separates $z(t^*) = x(t^*) + \psi(t^*)$, where $x(t^*)$ represents the slow dynamics while $\psi(t^*)$ is the fast oscillating term: $\psi(t^*) = \psi_0 \cos(2\tilde{\Omega} t^* + \varphi_0)$ with $\psi_0 = \Gamma^*[1 - \eta \mathrm{sinc}(k\pi)]/(2\tilde{\Omega}\sqrt{4\tilde{\Omega}^2 + \delta^2})$, $\varphi_0 = \Phi - \arctan(2\tilde{\Omega}/\delta)$. On averaging out $\psi(t^*)$ over time, the slow reduced dynamics of the kink becomes

$$\frac{d^2x}{dt^{*2}} + J_0(\psi_0) \sin z = -\delta \frac{dx}{dt^*} - \Gamma^* \eta \sin(\tilde{\Omega} t^*), \qquad (6)$$

where $J_0(\psi_0)$ is a zero-order Bessel function. Thus, the asymptotic kink dynamics when $\Omega \to \Omega_{\mathrm{PN}}$ could well be described by Eq. (6), which indicates that DKT disappears since the relevant symmetries are effectively restored [22]. This scenario is coherent with the gradual decrease of DKT strength as $\Omega \to \Omega_{\mathrm{PN}}$ (cf. Figs. 3 and 4), i.e., as the symmetries are *gradually* restored [20] (see supplementary material [23]).

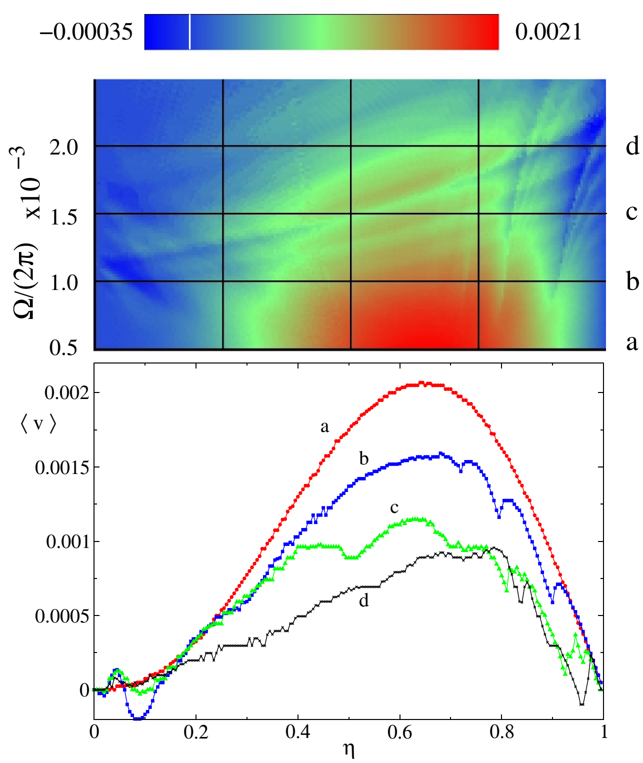

FIG. 3 (color online). Average kink velocity in the parameter plane $\eta - \Omega$ in the absence of disorder ($k = 0$) (top panel), and the detail of its dependence on η for the four labeled values of Ω (bottom panel). The fixed parameters are the same as in Fig. 1.

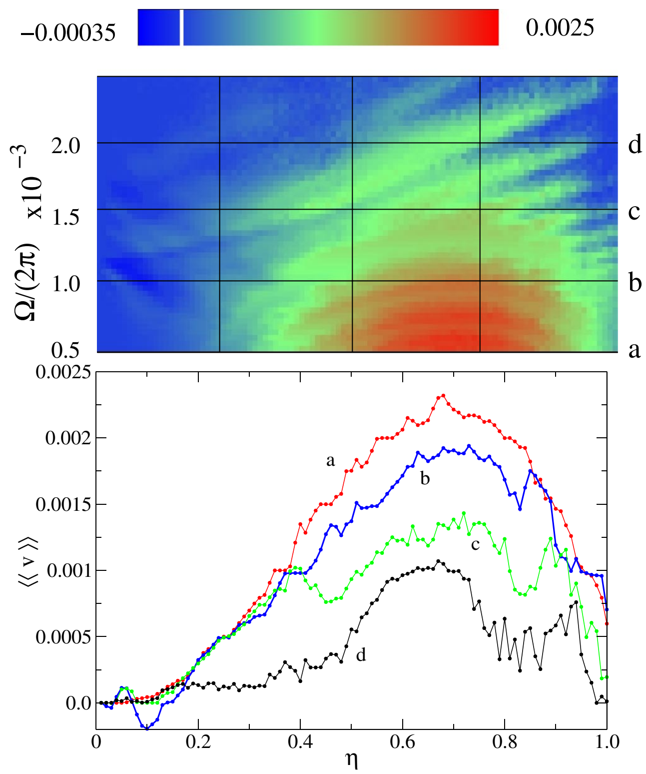

FIG. 4 (color online). Average kink velocity in the parameter plane $\eta - \Omega$ for $k = 0.2$ (top panel) and detail of its dependence on η for the four labeled values of Ω (bottom panel). The fixed parameters are the same as in Fig. 1.

In sum, the effectiveness of phase disorder in controlling directed transport of topological solitons induced by symmetry breaking of temporal forces in driven FK chains has been demonstrated. The general significance of the present results is that quenched temporal disorder may effectively act as an intrinsic organizational mechanism giving rise to different emergent phenomena, such as order-chaos transitions [19] and suppression/enhancement of directed transport. We expect that this effectiveness of the phase disorder can be extended to a number of phenomena, such as propagation of signals, absolute negative mobility, and general soliton ratchets. In a more general context, such as biological motors, the present disorder-induced ratchet mechanism could shed some light on the ratchetlike dynamics of sufficiently large populations of many such motors, since the phase of their internal drivers can in those cases be regarded as a random parameter. In addition, the present results provide unexpected additional support to the conjecture of one of the authors (R. Ch.) concerning directed transport by symmetry breaking of temporal forces [20].

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