Discrete Breathers in Two-Dimensional Josephson-Junction Arrays

J. J. Mazo
Departamento de Física de la Materia Condensada, Universidad de Zaragoza, E-50009 Zaragoza, Spain
Departamento de Teoría y Simulación de Sistemas Complejos, Instituto de Ciencia de Materiales de Aragón, C.S.I.C.–Universidad de Zaragoza, E-50009 Zaragoza, Spain
(Received 13 March 2002; published 18 November 2002)

We have proposed theoretically and studied numerically the existence of discrete breathers (intrinsic localized modes) in the dynamics of a two-dimensional Josephson-junction array biased by radio-frequency fields. The solutions are linearly stable in the framework of the Floquet theory and robust in the presence of thermal fluctuations. We have also discussed the conditions for realizing an experimental detection of these modes.

DOI: 10.1103/PhysRevLett.89.234101 PACS numbers: 05.45.Yv, 74.50.+r

Two-dimensional Josephson-junction arrays (2DJJA) are paradigmatic experimental systems for the study of many physical phenomena [1]. As realizations of the *XY* model, they have been designed for the study of phase transitions in unfrustrated and frustrated two-dimensional systems. Since they are tailored arrays, they have helped us to understand the role of geometry and disorder in granular superconductors. Modeled by coupled pendula, they are important to understand problems of synchronization of oscillators in complex lattices. Since they present vortices and antivortices, from these arrays we have learned from the behavior of those types of nonlinear coherent excitations in the presence of ac and/or dc perturbations and their role in equilibrium and nonequilibrium phase transitions.

A different type of coherent localized excitations in nonlinear lattices are the so-called discrete breathers (DB's) or intrinsic localized modes [2,3]. Physically they are dynamical solutions for which energy remains sharply localized in a few sites of the array. Then, there is not significant radiation of this energy to the rest of the lattice. It is important to emphasize that we are considering perfect and ordered homogeneous systems, the localization being an intrinsic property of such systems.

DB's have been mainly studied in lattices in one dimension and only experimentally found in some quasi-one-dimensional systems. One of such systems is an underdamped JJ ladder array biased by dc external currents. In these superconducting networks, the localized states are localized voltage solutions: not all of the junctions have the same voltage although they are all coupled and driven by the same current. Following theoretical predictions [4–6], DB's in the ladder were shown to exist for a wide range of parameter values, excited at will and detected by local voltage measurements and a low temperature laser scanning microscopy [7–10].

An interesting issue to address is the role and the possible detection of such excitations in the dynamics of 2D lattices. In this Letter we study this problem and show numerical evidence for the existence of DB's in the ac dynamics of a 2DJJA. We propose this system as an adequate device to carry out the experimental detection of DB's in a 2D system. In the array, the localized solution corresponds to the voltage localized solution sketched in Fig. 1. Such a nontrivial solution is driven by external currents and, in spite of the numerous studies on the dynamics of 2DJJA, to our knowledge it has been neither detected nor predicted up to the date.

In the classical regime, single JJ's can be modeled by the resistively and capacitively shunted junction RCSJ model. The normalized current through a junction is given by

$$i = \mathcal{N}(\varphi) = \ddot{\varphi} + \Gamma\dot{\varphi} + \sin\varphi. \tag{1}$$

In this equation φ is the gauge invariant phase difference across the junction. The damping is given by $\Gamma = \sqrt{\Phi_0/2\pi I_c C R^2}$, where Φ_0 is the flux quantum and I_c, C,

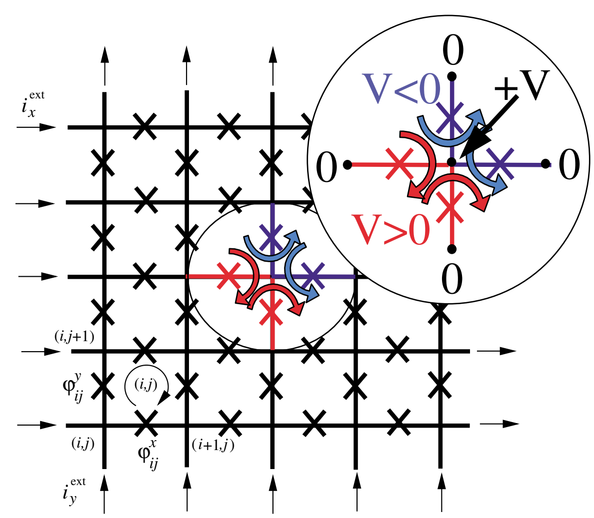

FIG. 1 (color online). Sketch of a DB, a voltage localized solution, in a 2DJJA. Junctions are represented by crosses. The dc voltage across any junction is zero except across four of them where it is equal to $+V$ or $-V$.

 0031-9007/02/89(23)/234101(4)$20.00 © 2002 The American Physical Society

and R are the junction critical current, capacitance, and resistance, respectively. Current has been measured in units of I_c and time in units of $\omega_p^{-1} = \sqrt{\Phi_0 C/2\pi I_c}$. The dc voltage across the junction is $\langle V \rangle = \frac{\Phi_0}{2\pi}\omega_p \langle \dot{\varphi} \rangle$.

The equations for the 2D array (Fig. 1) are derived by application of current conservation and fluxoid quantization laws. Then, for the simplest case of considering only self-induced magnetic fields and zero external field we get

$$\mathcal{N}(\varphi_{ij}^x) = \frac{\lambda}{h}(\xi_{ij} - \xi_{ij-1}) + i_x^{\text{ext}}/h, \qquad (2)$$
$$\mathcal{N}(\varphi_{ij}^y) = \lambda(\xi_{i-1j} - \xi_{ij}) + i_y^{\text{ext}}.$$

Here $\mathcal{N}(\varphi)$ stands for the nonlinear pendulum operator defined by Eq. (1). The anisotropy of the array h is the ratio of areas of the horizontal to vertical junctions, thus $h = A_x/A_y = I_{cx}/I_{cy} = C_x/C_y = R_y/R_x$. As defined above, current is measured in units of I_{cy} and λ controls the penetration depth of magnetic fields, $\lambda = \Phi_0/2\pi L I_{cy}$, with L the self-inductance of each plaquette [11].

ξ_{ij} is the flux of the magnetic field through the plaquette ij. In our case, all contributions to this flux come from self-induced magnetic fields, f_{ij}^{ind}. Such fields are related to the gauge invariant phase differences through fluxoid quantization conditions. Thus

$$\xi_{ij} = -2\pi f_{ij}^{\text{ind}} = \varphi_{ij}^y + \varphi_{ij+1}^x - \varphi_{i+1j}^y - \varphi_{ij}^x. \qquad (3)$$

The equations are completed with appropriate open boundary conditions.

It is physically interesting to define two λ parameters, one for each direction: $\lambda_x = \Phi_0/2\pi L I_{cx} = \lambda/h$ and $\lambda_y = \Phi_0/2\pi L I_{cy} = \lambda$. By writing Eqs. (2) in terms of λ_x and λ_y we see that coupling between junctions is controlled by these λ's.

Equations (2) represent a set of nonlinear oscillators with every phase coupled to some neighbor phases. A method which has been proved to be very useful for the theoretical and numerical analysis of localized solutions in coupled nonlinear oscillator systems is the continuation of solutions from the anticontinuous limit. The basic idea is easy to follow: if the system possesses a well defined uncoupled limit, where localized solutions are trivial solutions, then such solutions are generally safely continued from this limit.

In our system, $\lambda \to 0$ is an appropriate uncoupled limit where all the junctions behave as forced and damped isolated pendula [12].

Figure 2 shows two examples of IV curves of single underdamped junctions in the presence of ac external currents $i^{\text{ext}} = i_{\text{ac}} \sin\omega t$. We see that for some values of the parameters solutions coexist with dc voltage synchronized with the frequency of the external field, $\langle V \rangle = (n/m)\frac{\Phi_0}{2\pi}\omega$ [13]. This characteristic of the dynamics of underdamped pendula biased by ac fields [14] allows one to intuitively understand the localized

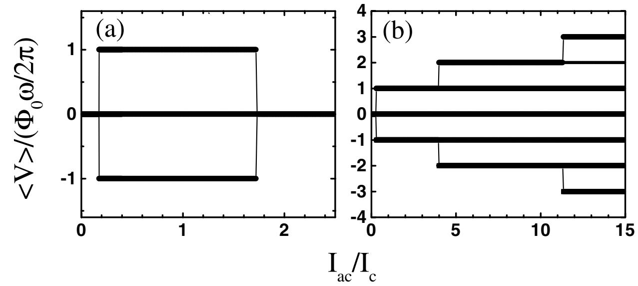

FIG. 2. The dc voltage, $\langle V \rangle$, versus current amplitude I_{ac} for a *single* JJ model biased by an ac current and for two values of damping and frequency, as well as different initial conditions. (a) $\Gamma = 0.02$ and $\omega/\omega_p = 2\pi \times 0.25$. Three different voltage states coexist for the same range of the amplitude of the external current. (b) $\Gamma = 0.003$ and $\omega/\omega_p = 2\pi \times 0.5$. Depending on the initial conditions we find the coexistence of many different voltage states. We show only harmonic solutions [$\langle V \rangle = (n/m)\frac{\Phi_0}{2\pi}\omega$ with $m = 1$].

solutions sketched in Fig. 1, where junctions with three values of dc voltage (0 or $\pm V$) also coexist.

In what follows we will present different situations where we have obtained DB's in 2DJJA. Simulations were done mostly in 11×11 arrays and exceptionally in 25×25 and 100×100 arrays using a 4th order Runge-Kutta integration scheme. Localized solutions are usually time periodic, and we have done Floquet linear stability analysis of the solutions and checked their robustness against thermal fluctuations. Our equations have five independent parameters. Most of the simulations were done at $h = 1$, the values of Γ, i_{ac}, and ω were obtained from simulations of the dynamics of single JJ's (similar to those shown in Fig. 2), and different values of λ were tried.

We start with isotropic arrays, $h = 1$. The first case corresponds to an array with diagonal bias [(11) direction], $i_x^{\text{ext}} = i_y^{\text{ext}} = i_{ac} \sin\omega t$. We have used DB solutions at the uncoupled limit as trial initial conditions for small values of λ. This has been done for different values of the damping and bias current, and in all cases DB's were found to be stable solutions of the dynamics at small values of λ. We will start showing numerical simulations for $\Gamma = 0.003$, $\omega/\omega_p = 2\pi \times 0.5$, and $i_{\text{ac}} = 5.0$. In this case, we were able to continue the DB solution up to $\lambda \simeq 0.2$ from the small λ limit.

Figure 3 shows a vector plot for a DB in an array biased by currents applied in the (11) direction at $h = 1$ and $\lambda = 0.1$. Vectors represent phases of the JJ's in the central region of an 11×11 array. Images from 3(a)–3(d) correspond to four different time instants along a period of the external force. As explained, the DB corresponds to a solution for which the four junctions around a common (central) island are in a resistive branch with voltage synchronized to the frequency of the bias current. In this case $m = n = 1$. Far from the DB core, junctions follow the external bias describing periodic oscillations.

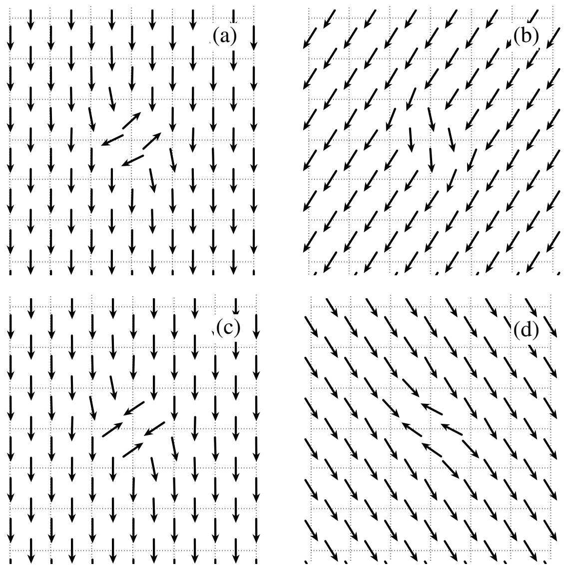

FIG. 3. Four snapshots of the phase evolution for a DB solution along a period of the solution. Current is biased in the (11) direction, $\lambda = 0.1$, $h = 1.0$, $I_{ac}/I_c = 5.0$, $\omega/\omega_p = 2\pi 0.5$, $\Gamma = 0.003$. We show the central part of an 11×11 array. Time increases from (a) to (d). We see that most phases merely oscillate (swing) about downward vertical, but the center four rotate, two clockwise and two counterclockwise.

As can be seen in the picture, the perturbation caused by the DB is highly localized and affects only a small number of junctions in the array. If we focus on the core of the breather, it shows a perfect left-right symmetry with respect to the current bias direction. We have done the Floquet stability analysis of the solution and checked the robustness of the localized state against fluctuations and small perturbations by integrating the equations of motion of the array with the inclusion of a small noisy current in the junction equations. Simulations show that the DB's are linearly stable periodic orbits and are also stable against thermal fluctuations for moderate strength of the fluctuations.

By *quasistatically* varying our parameters in the simulation, we have checked that localized solutions are also stable when decreasing i_{ac} to values close to 1 or when increasing the damping to values of Γ above 0.02. The Floquet analysis of the solutions has shown that when increasing Γ (also when decreasing i_{ac}) the mode responsible for the loss of stability of the solution is localized in the breather solution. However, when increasing the value of λ weak instabilities associated with extended eigenvectors appear. Such weak instabilities are typical of finite systems and weaken when the size of the system increases [15]. They are responsible for the destabilization of the localized solution observed, only after integration of the solution for thousands of periods of the external force, at large enough values of λ.

When the DB destabilizes, different final configurations have been observed. For instance, by increasing the damping the array usually decays into a coherent state where all the junctions librate uniformly and follow the external field. However, by increasing λ usually a chaotic solution of the whole array is found. In some cases other solutions appear.

DB's were also easily found in other regions of our five-dimensional parameter space. For instance, they were excited at $h = 1$ and small values of λ for $i_{ac} = 0.7$, $\omega = 2\pi 0.25$, and $\Gamma = 0.02$ [parameters as those in Fig. 2(a)].

We also obtained DB's for very large values of λ. In this case no continuation technique from any limit was used and the solutions were excited by playing with adequate initial conditions. The $\lambda \to \infty$ limit physically corresponds to neglect induced fields in the system. As this limit involves a drastic reduction of the number of variables of the system (roughly from $2N^2$ to N^2), it is a limit not trivial to study from Eqs. (2).

We present now the case of bias current along one of the main directions of the array. Let $i_x^{\rm ext} = 0$ and $i_y^{\rm ext} \neq 0$, for instance [(01) direction bias]. Again, the $\lambda \to 0$ limit is an appropriate limit to build localized solutions. It is worth mentioning that in contrast to the cases described above, now junctions along the current injection direction behave as forced and damped independent pendula, whereas the junctions perpendicular to this direction are unforced. Then, once switched on, the coupling is responsible for the rotation of horizontal junctions in the core of the breather. In these cases DB's were obtained at parameter values similar to those used above, and they were also found to be Floquet and thermal stable. We found again the existence of a mirror symmetry in the solutions ("the mirror" in this case follows the bias direction and crosses the core of the DB).

We will finish studying a different anticontinuous limit in Eqs. (2) which corresponds to the case of small I_{cx} (highly anisotropic array, $h \ll 1$; or $\lambda_x \gg \lambda_y$ for a given value of λ_y) and current in the (01) direction [16]. Then currents through x junctions are small, and the same is true for mesh currents and induced fields. It can be seen that this limit corresponds to negligible small ξ_{ij} for all ij and thus the junctions in the bias direction are effectively uncoupled. Now we expect to obtain localized solutions at small values of the anisotropy parameter h. In our simulations we have started exciting a DB solution at small values of h and continued it up to larger values of h until a destabilization point is reached. We have observed that in this case the localized solution destabilizes at small values of the anisotropy parameter h, typically $h < 0.1$ (the value also depends on the other chosen parameters: Γ, λ, i_{ac}, and ω). Before destabilization, this solution has also been numerically proved to be stable against thermal fluctuations in the system.

We have also studied the effect of external magnetic fields on the array. Such fields can be easily included in our model by defining $\xi_{ij} = -2\pi(f_0 + f_{ij}^{\rm ind})$ where f_0 is the flux of the external magnetic field measured in units of the flux quantum. Simulations of the system equations combined with Floquet stability analysis show that DB states also exist as attractors of the dynamics in the presence of external magnetic fields.

There is much work which still can be done on the subject. For instance, new values of the current parameter have to be studied. Existence regions for the localized solutions depend very strongly on these parameters, and thus a systematic study needs to be done. Also a more systematic analysis of the normal mode frequencies for the array [17] and Floquet stability analysis will help one to understand the occurring different bifurcation scenarios. The role of resonances between the DB and electromagnetic waves or the effect of the size of the system are other aspects to consider.

The fact that many DB's solutions present a mirror symmetry combined with the localized character of the solutions allows for proposing a single plaquette as a reduced model for studying the behavior of larger arrays. Such drastic reduction of the dimensionality of the problem has been successfully realized for the analysis of DB's in Josephson ladders [6,18]. There, the single plaquette has been proved to be useful for understanding different points of this complex nonlinear dynamics problem.

Another topic to discuss is the experimental detection of such modes. This will be an important challenge for the experimentalists in the field. Arrays with the desired parameters can be easily made, and the external driving is similar to that used in standard voltage designs. There exist, however, at least three difficulties that still remain to be discussed. The first considered is the feasibility of having a uniform driving in the array as is supposed in the simulations. A second aspect is related to the operations to excite the DB's in the array. We are looking for a controlled excitation protocol for DB's in 2D arrays. A possibility we propose is to add a local dc current that should be injected in one central island of the array and extracted from the four neighboring islands. Using this idea we have numerically excited DB solutions in the array. First we used the local dc injection to carry four junctions to rotate. Then we applied the ac field increasing the amplitude of the signal slowly from zero. We finished by decreasing to zero the local dc bias. In many cases the method resulted in a DB solution. The last issue is the experimental detection of the mode. This may be done by the use of local voltage probes and, if possible, a low temperature scanning laser microscopy [8].

I finish with some words about the possible role of intrinsic localized excitations in approach to equilibrium problems in 2D arrays. In many cases, DB solutions can be seen as bounded kink-antikink or vortex-antivortex pairs [2–4,10]. An important issue to study is the possible spontaneous thermal activation of localized solutions [19], in general, 2D arrays. Then the role that this type of excitations would play in the out of equilibrium properties of 2D arrays is an issue to explore, in particular, if DB's play any role in the nucleation of vortex-antivortex pairs, which drives phase transitions of the array.

The author thanks F. Falo, M.V. Fistul, S. Flach, L. M. Floría, J. L. García-Palacios, P. J. Martínez, T. P. Orlando, F. Pignatelli, M. Schuster, K. Segall, E. Trías, and A.V. Ustinov for insightful discussions. Financial support of DGES PB98-1592, BFM 2002-00113, European Network LOCNET HPRN-CT-1999-00163, and Fulbright Commission is acknowledged.

[1] R. S. Newrock, C. J. Lobb, U. Geigenmüller, and M. Octavio, Solid State Phys. **54**, 263 (2000).
[2] S. Aubry, Physica (Amsterdam) **103D**, 201 (1997).
[3] S. Flach and C. R. Willis, Phys. Rep. **295**, 181 (1998).
[4] L. M. Floría, J. L. Marín, P. J. Martínez, F. Falo, and S. Aubry, Europhys. Lett. **36**, 539 (1996).
[5] S. Flach and M. Spicci, J. Phys. Condens. Matter **11**, 321 (1999).
[6] J. J. Mazo, E. Trías, and T. P. Orlando, Phys. Rev. B **59**, 13 604 (1999).
[7] E. Trías, J. J. Mazo, and T. P. Orlando, Phys. Rev. Lett. **84**, 741 (2000).
[8] P. Binder, D. Abraimov, A.V. Ustinov, S. Flach, and Y. Zolotaryuk, Phys. Rev. Lett. **84**, 745 (2000).
[9] P. Binder, D. Abraimov, and A.V. Ustinov, Phys. Rev. E **62**, 2858 (2000).
[10] E. Trías, J. J. Mazo, A. Brinkman, and T. P. Orlando, Physica (Amsterdam) **156D**, 98 (2001).
[11] A. Majhofer, T. Wolf, and W. Dieterich, Phys. Rev. B **44**, 9634 (1991).
[12] The $\lambda = 0$ limit deserves to be studied carefully. We have monitored the evolution of the $\lambda\xi_{ij}$ products when decreasing λ and numerically checked that they go to zero as λ does.
[13] Note that at a fixed value of I_{ac} each possible voltage value corresponds to a trace back to $I_{\mathrm{dc}} = 0$ of current biased Shapiro steps of underdamped junctions. This result has been successfully exploited to design voltage standards based in series arrays of such junctions.
[14] Underdamped JJ's biased by dc currents support two coexisting solutions at the most.
[15] J. L. Marín and S. Aubry, Physica (Amsterdam) **119D**, 163 (1998).
[16] We could also study (10) bias and the large h case.
[17] For an array with periodic boundary conditions the frequencies of the linear waves are easy to find, and they are bounded by $1 < \omega_{\mathrm{res}} < \sqrt{1 + 8\lambda}$. Frequencies for the open boundary problem are slightly different. We see that the frequencies of most of the DB's studied in this work are well above these bounds.
[18] A. Benabdallah, M.V. Fistul, and S. Flach, Physica (Amsterdam) **159D**, 202 (2001).
[19] G. P. Tsironis and S. Aubry, Phys. Rev. Lett. **77**, 5225 (1996); M. Peyrard, Physica (Amsterdam) **119D**, 184 (1998); A. Bikaki, N. K. Voulgarakis, S. Aubry, and G. P. Tsironis, Phys. Rev. E **59**, 1234 (1999).